

The Theory of Nuclear Forces: Is the Never-Ending Story Coming to an End?

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I review the recent progress in our understanding of nuclear forces in terms of an effective field theory (EFT) for low-energy QCD and put this progress into historical perspective. This is followed by an assessment of the current status of EFT based nuclear potentials. In concluding, I will summarize some unresolved issues.

1. Historical Perspective

The theory of nuclear forces has a long history. Based upon the Yukawa idea [1] and the discovery of the pion, the 1950's became the first period of "pion theories". These, however, resulted in failure—for reasons we understand today: pion dynamics is ruled by chiral symmetry, a constraint that was not realized in the theories of the 1950's.

The 1960's and 70's represent the main period for theories that also include heavy mesons ("meson theories") [2], but the work on meson models continued all the way into the 1990's when the family of the so-called high-precision NN potentials was developed. This family includes the Nijm-I, Nijm-II, and Reid93 potentials [3], the Argonne V_{18} [4], and the CD-Bonn potential [5, 6]. Later, also the highly non-local potential by Doleschall *et al.* [7] and the "CD-Bonn + Δ " model by Deltuva *et al.* [8] joined the club. All these potentials have in common that they are charge-dependent, use about 40-50 parameters, and reproduce the 1992 Nijmegen NN data base with a $\chi^2/\text{datum} \approx 1$.

Over the past ten years, the high-precision potentials have been applied intensively in exact few-body calculations and microscopic nuclear structure theory. Already the first few applications of the high-precision potentials in three-nucleon reactions [9] clearly revealed sizable differences between the predictions from NN potentials with a $\chi^2/\text{datum} \approx 1$ (i.e., the new generation of potentials) and NN potentials with a $\chi^2/\text{datum} \approx 2$ (the old generation of the 1970's and 80's which includes the old Nijmegen, the Paris, and the old Bonn potentials). Thus, once for all, the standard of precision was established that must be met by any future work in microscopic nuclear structure and exact few-body calculations: The input NN potential must reproduce the NN data with a $\chi^2/\text{datum} \approx 1$ or the uncertainty in the predictions will make it impossible to draw reliable conclusions.

In spite of these great practical achievements, the high-precision potentials cannot be the end of the story, because they are all phenomenological in nature. Ultimately, we need potentials that are based on proper theory *and* yield quantitative results.

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Table 1

χ^2/datum for the reproduction of the 1999 np database by families of np potentials at NLO and NNLO constructed by the Bochum/Juelich group [15].

Bin (MeV)	# of np data	<i>Bochum/Juelich</i>	
		NLO	NNLO
0–100	1058	4–5	1.4–1.9
100–190	501	77–121	12–32
190–290	843	140–220	25–69
0–290	2402	67–105	12–27

Since the nuclear force is a manifestation of strong interactions, any serious derivation has to start from quantum chromodynamics (QCD). However, the well-known problem with QCD is that it is non-perturbative in the low-energy regime characteristic for nuclear physics. For many years, this fact was perceived as a major obstacle for a derivation of the nuclear force from QCD—impossible to overcome except by lattice QCD. The effective field theory (EFT) concept has shown the way out of this dilemma. One has to realize that the scenario of low-energy QCD is characterized by pions and nucleons interacting via a force governed by spontaneously broken approximate chiral symmetry.

Based upon this EFT, a systematic expansion in terms of $(Q/\Lambda_\chi)^\nu$ can be developed, where Q denotes a momentum or pion mass, $\Lambda_\chi \approx 1$ GeV is the chiral symmetry breaking scale, and $\nu \geq 0$ [10]. This has become known as chiral perturbation theory (ChPT). In contrast to the pion theories of the 1950’s, this new chirally constrained “pion theory” is quite successful. Starting with the pioneering work by Ordóñez, Ray, and van Kolck in the early 1990’s [11], NN potentials have been constructed in the framework of ChPT. In the next section, I will discuss the latest status of this work.

A remarkable fact of ChPT is that it makes specific predictions also for many-body forces. For a given order of ChPT, two-nucleon forces (2NF), three-nucleon forces (3NF), ... are generated on the same footing (see Ref. [12] for a concise review). At leading order (LO), there are no 3NF, and at next-to-leading order (NLO), all 3NF terms cancel [10, 13]. However, at next-to-next-to-leading order (NNLO) and higher orders, well-defined, nonvanishing 3NF occur [13, 14]. Since 3NF show up for the first time at NNLO, they are weak. Four-nucleon forces (4NF) occur first at N³LO and are, therefore, even weaker.

2. Current status of chiral NN potentials

As pointed out in the previous section, the research conducted during the high-precision period of the 1990’s [9] systematically tested and firmly established the standards of precision that must be met by NN potentials to yield sufficiently accurate predictions when applied in microscopic calculations of few-body systems. Since in nuclear EFT we are dealing with a perturbative expansion (ChPT), the question is to what order of ChPT we have to go to obtain the precision that meets those standards. To discuss this issue on firm grounds, I show in Table 1 the χ^2/datum for the fit of the world np data below 290 MeV for a family of np potentials at NLO and NNLO. The NLO potentials produce the horrendous χ^2/datum between 67 and 105, and the NNLO are between 12 and 27.

Table 2

χ^2 /datum for the reproduction of the 1999 ***np*** database by various *np* potentials. Numbers in parentheses denote cutoff parameters in units of MeV.

Bin (MeV)	# of <i>np</i> data	<i>Idaho</i> N ³ LO [18] (500–600)	<i>Bochum/Juelich</i> N ³ LO [19] (600/700–450/500)	Argonne V_{18} [4]
0–100	1058	1.0–1.1	1.0–1.1	0.95
100–190	501	1.1–1.2	1.3–1.8	1.10
190–290	843	1.2–1.4	2.8–20.0	1.11
0–290	2402	1.1–1.3	1.7–7.9	1.04

The rate of improvement from one order to the other is very encouraging, but the quality of the reproduction of the *np* data at NLO and NNLO is obviously totally insufficient for reliable predictions. In the literature, one can find calculations employing NLO and NNLO potentials in few-nucleon systems. Because of the extremely poor accuracy of such potentials, no reliable conclusions can be drawn from such calculations, rendering these applications essentially useless.

Based upon these facts, it has been pointed out in 2002 by Entem and Machleidt [16, 17] that NNLO is insufficient and one has to proceed to N³LO. Consequently, the first N³LO potential was created in 2003 [18], which showed that at this order a χ^2 /datum comparable to the high-precision Argonne V_{18} potential can, indeed, be achieved, see Table 2. This “Idaho” N³LO potential [18] produces a χ^2 /datum = 1.1 for the world *np* data below 290 MeV which compares well with the χ^2 /datum = 1.04 by the Argonne potential. In 2005, also the Bochum/Juelich group produced several N³LO NN potentials [19], the best of which fits the *np* data with a χ^2 /datum = 1.7 and the worse with a χ^2 /datum = 7.9 (see Table 2). While 7.9 is clearly unacceptable for any meaningful application, a χ^2 /datum of 1.7 is reasonable, although it does not meet the precision standard established in the 1990’s.

I turn now to the *pp* data. Typically, χ^2 for *pp* data are larger than for *np* because of the higher precision of *pp* data. Thus, the Argonne V_{18} produces a χ^2 /datum = 1.4 for the world *pp* data below 290 MeV and the best Idaho N³LO *pp* potential obtains 1.5. The fit by the best Bochum/Juelich N³LO *pp* potential results in a χ^2 /datum = 2.9

Table 3

χ^2 /datum for the reproduction of the 1999 ***pp*** database by various *pp* potentials. Numbers in parentheses denote cutoff parameters in units of MeV.

Bin (MeV)	# of <i>pp</i> data	<i>Idaho</i> N ³ LO [18] (500–600)	<i>Bochum/Juelich</i> N ³ LO [19] (600/700–450/500)	Argonne V_{18} [4]
0–100	795	1.0–1.7	1.0–3.8	1.0
100–190	411	1.5–1.9	3.5–11.6	1.3
190–290	851	1.9–2.7	4.3–44.4	1.8
0–290	2057	1.5–2.1	2.9–22.3	1.4

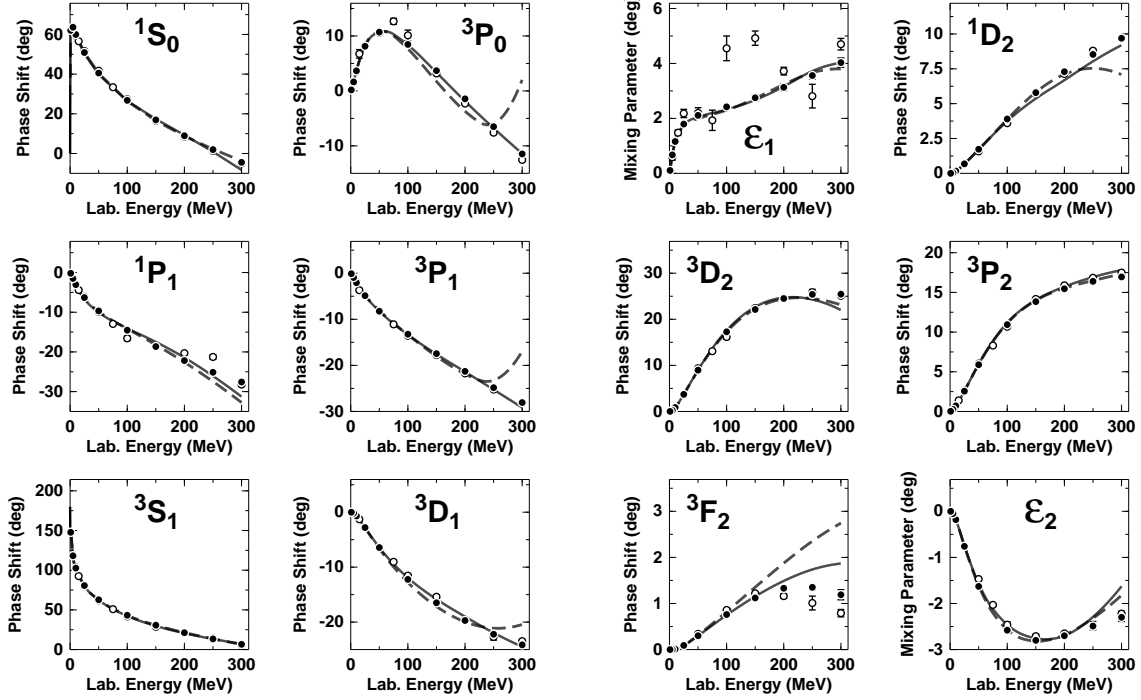


Figure 1. np phase shifts for energies below 300 MeV and partial waves with total angular momentum $J \leq 2$. The solid and the dashed curves show the phase shifts predicted by the N³LO potentials constructed by the Idaho [18] and the Bochum/Juelich [19] groups, respectively. Solid dots and open circles represent the Nijmegen multienergy np phase shift analysis [20] and the GWU/VPI single-energy np analysis SM99 [21], respectively.

which, again, is not quite consistent with the precision standards of the 1990's. The worst Bochum/Juelich N³LO pp potential produces a χ^2/datum of 22.3 and is incompatible with reliable predictions.

Phase shifts of np scattering from the best Idaho (solid line) and Bochum/Juelich (dashed line) N³LO np potentials are shown in Figure 1. The phase shifts confirm what the corresponding χ^2 's have already revealed.

3. The low-energy constants

The two-pion-exchange contribution to the NN interaction depends on the low-energy constants (LECs) of the dimension-two Lagrangian, commonly denoted by c_i , and the dimension-three LECs, d_i [17]. These parameters have been determined in πN data analyses, see column πN of Table 4. The most influential LEC is c_3 and, so, we will focus here just on c_3 . Not surprisingly, the Idaho and Bochum/Juelich N³LO potentials apply essentially the same value, namely, $c_3 = -3.3 \pm 0.1 \text{ GeV}^{-1}$, which is on the lower side but still roughly within one standard deviation of its πN determination. There exists also a determination of this parameter from the NN data by the Nijmegen group. They report $c_3 = -5.08 \pm 0.28 \text{ GeV}^{-1}$ in their 1999 pp analysis [24] and $c_3 = -4.78 \pm 0.10$

Table 4

Low-energy parameters. c_i are given in units of GeV^{-1} and \bar{d}_i in units GeV^{-2} .

	πN [22, 23]	<i>Idaho</i> [18] N ³ LO pot.	<i>Boch./J.</i> [19] N ³ LO pot.	Nijmegen 2003 PWA [25] (1999 PWA [24])
c_1	-0.81 ± 0.15	-0.81	-0.81	-0.76 (-0.76)
c_2	3.28 ± 0.23	2.80	3.28	- (-)
c_3	-4.69 ± 1.34	-3.20	-3.40	-4.78 ± 0.10 (-5.08 ± 0.28)
c_4	3.40 ± 0.04	5.40	3.40	3.96 ± 0.22 (4.70 ± 0.70)
$d_1 + d_2$	3.06 ± 0.21	3.06	3.06	-
\bar{d}_3	-3.27 ± 0.73	-3.27	-3.27	-
\bar{d}_5	0.45 ± 0.42	0.45	0.45	-
$\bar{d}_{14} - \bar{d}_{15}$	-5.65 ± 0.41	-5.65	-5.65	-

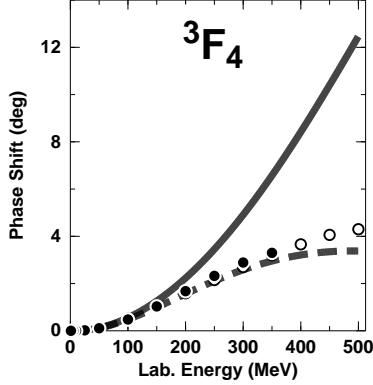


Figure 2. 3F_4 phase shifts calculated at NNLO using $c_3 = -4.78 \text{ GeV}^{-1}$. The solid curve is based upon the NNLO amplitude without cutoff, while for the dashed curve the cutoff used in the Nijmegen analysis [25] is employed. Solid dots and open circles like in Fig. 1.

GeV^{-1} in their 2003 $pp+np$ analysis [25]. Knowing the sensitivity of NN phase shifts and observables to c_3 , the difference between the NN potential values of about -3.3 GeV^{-1} and the Nijmegen NN analysis value of -4.8 GeV^{-1} is *huge*. Off-hand, it is hard to understand how two different types of NN analyses can be in such severe disagreement.

However, the discrepancy is easily explained if one looks into the details of the Nijmegen analysis [26]. The Nijmegen group starts from the NN amplitude at NNLO in momentum space, Fourier transforms it into r -space to obtain a local potential, and then cuts this potential off at $r=1.6 \text{ fm}$ (i.e., the potential is set to zero for $r \leq 1.6 \text{ fm}$). The impact of this cutoff is demonstrated in Figure 2 for the 3F_4 phase shifts, which is a representative case. In that figure, the solid line shows the phase shifts derived from the NN amplitude at NNLO using $c_3 = -4.78 \text{ GeV}^{-1}$ and applying no cutoffs (i.e., the actual model-independent amplitude at NNLO is used). The dashed line is obtained when the Nijmegen cutoff is applied and, as before, $c_3 = -4.78 \text{ GeV}^{-1}$ is used. The difference between the two curves in Fig. 2 is entirely due to the cutoff applied in the Nijmegen

analysis. It is clearly seen that the influence of this cutoff is dramatic and introduces a huge systematic error rendering the Nimegen analysis and their results unreliable [26].

It must be noted that the Idaho and Bochum/Juelich N³LO potentials also use cutoffs, but not as harsh ones as in the Nijmegen analysis.

The best way to determine the LECs from NN data is to apply no cutoffs to the NN amplitudes (at NNLO or N³LO) in peripheral partial waves and to keep the amplitudes of lower partial waves at their empirical values (as determined in phase shift analysis). In such an analysis the value $c_3 = -2.7 \text{ GeV}^{-1}$ is obtained [27].

4. Conclusions

The theory of nuclear forces has made great progress since the turn of the millennium. Nucleon-nucleon potentials have been developed that are based on proper theory (EFT for low-energy QCD) and are of high-precision, at the same time. Moreover, the theory generates two- and many-body forces on an equal footing and provides a theoretical explanation for the empirically known fact that $2\text{NF} \gg 3\text{NF} \gg 4\text{NF} \dots$

At N³LO [17, 18], the accuracy can be achieved that is necessary and sufficient for microscopic nuclear structure. First calculations applying the N³LO NN potential [18] in the (no-core) shell model [28, 29, 30, 31, 32], the coupled cluster formalism [33, 34, 35, 36, 37], and the unitary-model-operator approach [38] have produced promising results.

The 3NF at NNLO is known [13, 14] and has been applied in few-nucleon reactions [14, 39, 40] as well as the structure of light nuclei [41, 42]. However, the famous ‘ A_y puzzle’ of nucleon-deuteron scattering is not resolved by the 3NF at NNLO. Thus, the most important outstanding issue is the 3NF at N³LO, which is under construction [43].

Another open question that needs to be settled is whether Weinberg power counting, which is applied in all current NN potentials, is consistent. This controversial issue is presently being debated in the literature [44, 45, 46].

It may be too early to claim that the never-ending-story is coming to an end, but the story seems to be converging—at the same rate as chiral perturbation theory.

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REFERENCES

1. H. Yukawa, Proc. Phys. Math. Soc. Japan **17**, 48 (1935).
2. R. Machleidt, Adv. Nucl. Phys. **19**, 189 (1989).
3. V. G. J. Stoks *et al.*, Phys. Rev. C **49**, 2950 (1994).
4. R. B. Wiringa *et al.*, Phys. Rev. C **51**, 38 (1995).
5. R. Machleidt, F. Sammarruca, and Y. Song, Phys. Rev. C **53**, 1483 (1996).
6. R. Machleidt, Phys. Rev. C **63**, 024001 (2001).
7. P. Doleschall, I. Borbély, Z. Papp, and W. Plessas, Phys. Rev. C **67**, 064005 (2003).
8. A. Deltuva, R. Machleidt, and P. U. Sauer, Phys. Rev. C **68**, 024005 (2003).
9. W. Glöckle *et al.*, Phys. Rep. **274**, 107 (1996); and W. Glöckle, private communication.

10. S. Weinberg, Phys. Lett. B **251**, 288 (1990); Nucl. Phys. **B363**, 3 (1991); Phys. Lett. B **295**, 114 (1992).
11. C. Ordóñez *et al.*, Phys. Rev. Lett. **72**, 1982 (1994); Phys. Rev. C **53**, 2086 (1996).
12. R. Machleidt and D. R. Entem, J. Phys. G: Nucl. Phys. **31**, S1235 (2005).
13. U. van Kolck, Phys. Rev. C **49**, 2932 (1994).
14. E. Epelbaum *et al.*, Phys. Rev. C **66**, 064001 (2002).
15. E. Epelbaum, W. Glöckle, and U.-G. Meißner, Eur. Phys. J. **A19**, 401 (2004).
16. D. R. Entem and R. Machleidt, Phys. Lett. B **524**, 93 (2002).
17. D. R. Entem and R. Machleidt, Phys. Rev. C **66**, 014002 (2002).
18. D. R. Entem and R. Machleidt, Phys. Rev. C **68**, 041001 (2003).
19. E. Epelbaum, W. Glöckle, and U.-G. Meißner, Nucl. Phys. A747, 362 (2005).
20. V. G. J. Stoks *et al.*, Phys. Rev. C **48**, 792 (1993).
21. SAID, solution SM99 (Summer 1999).
22. P. Büttiker *et al.*, Nucl. Phys. **A668**, 97 (2000).
23. N. Fettes *et al.*, Nucl. Phys. **A640**, 199 (1998).
24. M. C. M. Rentmeester *et al.*, Phys. Rev. Lett. **82**, 4992 (1999).
25. M. C. M. Rentmeester *et al.*, Phys. Rev. C **67**, 044001 (2003).
26. D. R. Entem and R. Machleidt, arXiv:nucl-th/0303017.
27. D. R. Entem and R. Machleidt, unpublished.
28. L. Coraggio *et al.*, Phys. Rev. C **66**, 021303 (2002).
29. L. Coraggio *et al.*, Phys. Rev. C **71**, 014307 (2005).
30. P. Navrátil and E. Caurier (2004) *Phys. Rev. C* **69** 014311.
31. C. Forssen *et al.*, Phys. Rev. C **71**, 044312 (2005).
32. J.P. Vary *et al.*, Eur. Phys. J. A **25** s01, 475 (2005).
33. K. Kowalski *et al.*, Phys. Rev. Lett. **92**, 132501 (2004).
34. D.J. Dean and M. Hjorth-Jensen (2004) *Phys. Rev. C* **69** 054320.
35. M. Wloch *et al.*, J. Phys. G **31**, S1291 (2005); Phys. Rev. Lett. **94**, 21250 (2005).
36. D.J. Dean *et al.*, Nucl. Phys. **A752**, 299 (2005).
37. J.R. Gour *et al.*, Phys. Rev. C **74**, 024310 (2006).
38. S. Fujii, R. Okamoto, and K. Suzuki, Phys. Rev. C **69**, 034328 (2004).
39. K. Ermisch *et al.*, Phys. Rev. C **71**, 064004 (2005).
40. H. Witala, J. Golak, R. Skibinski, W. Glöckle, A. Nogga, E. Epelbaum, H. Kamada, A. Kievsky, and M. Viviani, Phys. Rev. C **73**, 044004 (2006).
41. A. Nogga *et al.*, Nucl. Phys. **A737**, 236 (2004).
42. A. Nogga, P. Navratil, B.R. Barrett, and J.P. Vary, Phys. Rev. C **73**, 064002 (2006).
43. U.-G. Meißner, this proceedings.
44. A. Nogga, R. G. E. Timmermans, and U. van Kolck, Phys. Rev. C **72**, 054006 (2005).
45. E. Epelbaum, U.-G. Meißner, arXiv:nucl-th/0609037.
46. U. van Kolck, this proceedings.